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14. ABSTRACT
One-dimensional compressible flows of calorically perfect gases in which only a single driving potential is present are called sim flows[1, 2, 3]. Because Mach number and property relations for simple flows can be expressed in closed form, such flows are usua treated individually. Most introductory compressible flow courses and most compressible flow textbooks discuss three types of sim flows: isentropic flow in a variable area duct, the heat addition (or rejection) flow in constant area duct (Rayleigh flow), and adiaba flow in a constant area duct with gas-wall-surface friction (Fanno flow). A more complicated case of a simple mass addition flow described in literature[4]. In the present paper we consider a case of two driving potentials: heat addition and variable area du Therefore, the obtained governing ordinary differential equations cannot be solved analytically, as in the case of simple flows, but th can be easily solved numerically. This research has been inspired by recent papers[5, 6], where the authors discussed the thr augmentation of solid rocket motors by heating the alumina particles in the exhaust of the motor by a microwave beam generated b ground-based microwave generation facility. Thus, the equations obtained in the paper can be used, for example, as benchmarks numerical investigation of gas flows in a nozzle heated by an external heating source, such as in[5, 6].

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One-Dimensional Compressible Flow in Variable Area Duct with Heat Addition (Preprint)

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Nomenclature

A	= area of the duct
c_P	= specific heat of perfect gas per unit of mass at constant pressure
k	= Boltzmann constant
M	= Mach number
m	= mass of gas molecule
$\dot{m}$	= mass flux through the duct
n	= number density
P	= pressure
T	= temperature
u	= fluid velocity
x	= axis along the duct
Γ	= enthalpy per unit of mass at the duck inlet
γ	= gaseous constant
ε	= heat input per unit of mass
ρ	= mass density

I. Introduction

One-dimensional compressible flows of calorically perfect gases in which only a single driving potential is present are called simple flows[1, 2, 3]. Because Mach number and property relations for simple flows can be expressed in closed form, such flows are usually treated individually. Most introductory compressible flow courses and most compressible flow textbooks discuss three types of simple flows: isentropic flow in a variable area duct, the heat addition (or rejection) flow in constant area duct (Rayleigh flow), and adiabatic flow in a constant area duct with gas-wall-surface friction (Fanno flow). A more complicated case of a simple mass addition flow is described in literature[4]. In the present paper we consider a case of two driving potentials: heat addition and variable area duct. Therefore, the obtained governing ordinary differential equations cannot be solved analytically, as in the case of simple flows, but they can be easily solved numerically. This research has been inspired by recent papers[5, 6], where the authors discussed the thrust augmentation of solid rocket motors by heating the alumina particles in the exhaust of the motor by a microwave beam generated by a ground-based microwave generation facility. Thus, the equations

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obtained in the paper can be used, for example, as benchmarks for numerical investigation of gas flows in a nozzle heated by an external heating source, such as in[5, 6].

II. Theory

A set of equations describing a one-dimensional gas flow in a nozzle heated by an external heating source, Fig. 1, can be written as

$$\dot{m} = \rho \cdot u \cdot A \tag{1}$$

$$\rho \cdot u \cdot \frac{du}{dx} = -\frac{dP}{dx} \tag{2}$$

$$\frac{d\varepsilon}{dx} = \frac{d}{dx}\left(c_P \cdot T + \frac{u^2}{2} \right) \tag{3}$$

where the first, second, and the equations correspond to the laws of conservation of mass, momentum, and energy.

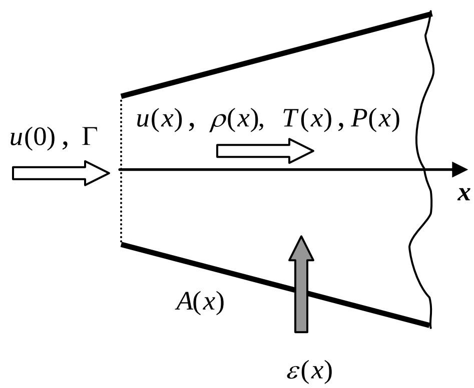

Fig. 1. Schematic of gas flow through the nozzle

Introducing the gas enthalpy per unit mass at the inlet of the duct, Fig. 1, we obtain from Eq. (3) that

$$\Gamma = c_P \cdot T + \frac{u^2}{2} - \varepsilon \tag{4}$$

Substituting

$$n = \frac{\dot{m}}{m \cdot u \cdot A} \quad \text{and} \quad T = \frac{1}{c_P} \cdot \left(\Gamma - \frac{u^2}{2} + \varepsilon \right) \tag{5}$$

where T is obtained from Eq. (4), into the equation for pressure $P = n \cdot k \cdot T$, we obtain

$$P = \frac{\dot{m} \cdot k}{m \cdot c_P \cdot u \cdot A} \cdot \left(\Gamma - \frac{u^2}{2} + \varepsilon \right) = \frac{\dot{m} \cdot (\gamma - 1)}{\gamma \cdot u \cdot A} \cdot \left(\Gamma - \frac{u^2}{2} + \varepsilon \right) \tag{6}$$

Here we have used γ for a perfect gas, i.e., $(\gamma - 1)/\gamma = k / m \cdot c_P$. Substituting Eq. (6) for P and $\rho \cdot u = \dot{m} / A$, Eq. (1), into Eq. (2) after simple algebra we obtain a governing equation for flow velocity u vs. x

$$\frac{d}{dx}\left[\left(\frac{\Gamma}{u} + u \cdot \frac{(\gamma + 1)}{2 \cdot (\gamma - 1)} + \frac{\varepsilon}{u} \right) \right] = \left(\frac{\Gamma}{u} - \frac{u}{2} + \frac{\varepsilon}{u} \right) \cdot \frac{d(\ln A)}{dx} \tag{7}$$

where $\gamma = 5/3$, Γ is an input constant and ε and A are input functions on x.

Let us obtain a governing equation for Mach number. Substituting T from Eq. (5) into the expression for Mach number

$$M = \frac{u}{\sqrt{(\gamma - 1) \cdot T \cdot c_P}} \tag{8}$$

we obtain

$$u = M \cdot \left(\frac{(\Gamma + \varepsilon) \cdot (\gamma - 1)}{1 + 0.5 \cdot \frac{\gamma - 1}{2} \cdot M^2} \right)^{1/2} \tag{9}$$

Substituting Eq. (9) into Eq. (7) after some algebra we obtain a governing equation for Mach number:

$$\left[\frac{1 - M^2}{M \cdot \left(1 + \frac{\gamma - 1}{2} \cdot M^2 \right)} \right] \cdot \frac{dM}{dx} = \frac{(1 + \gamma \cdot M^2)}{2} \cdot \frac{d(\ln[\varepsilon + \Gamma])}{dx} - \frac{d(\ln A)}{dx} \tag{10}$$

Unlike in the case of a simple flow, Eq. (10) generally cannot be solved in radicals; however, it can be solved numerically via standard methods. It can be shown that the governing equations for isentropic and Rayleigh flows (the cases of $\varepsilon = 0$ and $A = const$, respectively) can be derived from Eq. (10).

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References

[1]Shapiro, A. H., *The Dynamics and Thermodynamics of Compressible Fluid Flow*, Ronald Press, New York, 1953.

[2]Zucrow, M. J., and Hoffman, J. D., *Gas Dynamics*, Willey, New York, 1976.

[3]Saad, M. A., Compressible Fluid Flow, Prentice-Hall, Englewood Cliffs, New Jersey, 1987.

[4]Hodge, B. K. and Young, F., "Mass Addition Process in One-Dimensional Compressible Flow," *31st Aerospace Sciences Meeting & Exhibit*, Reno, NV, 11-14 January, 1993, AIAA 93-0487.

[5]Cornella, B., Hammerland, S., Ketsdever, A. D., and Song, H., "Power Generation for a Beamed Microwave Propulsion Concept," *45st Joint Propulsion Conference & Exhibit*, Denver, CO, 2-5 August, 2009, AIAA 2009-4872.

[6]Gimelshein, N. E., Gimelshein, S. F., P. and Ketsdever, A., "Thrust Augmentation in Solid Rocket Motor Using Beamed Microwave Energy," *45st Joint Propulsion Conference & Exhibit*, Denver, CO, 2-5 August, 2009, AIAA 2009-4962.